 **BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI**

**WORK INTEGRATED LEARNING PROGRAMMES**

**COURSE HANDOUT**

**Part A: Content Design**

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| **Course Title** | LINEAR ALGEBRA & OPTIMIZATION |
| **Course No(s)** | SEWP ZC132 / CSIW ZC132 |
|  | II sem 2018 -19 |

**Course Description**

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| Course Description: This course consists of topics from Linear Algebra and Optimization **Linear Algebra:** Operations on matrices, Linear system of equations, Vector spaces, Linear transformation, Eigenvalues and Eigenvectors.  **Optimization:** Modelling with Linear programming, Graphical approach, The Simplex method, Big M and Two Phase Method, Special cases of the simplex method and sensitivity analysis, The Dual Simplex Method and the Generalized Simplex Algorithm, Post optimal analysis. |

**Course Objectives**

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| **No** | **Objective** |
| **CO1** | Understand concepts in linear algebra and use it as a platform to model physical problems. |
| **CO2** | Appreciate analytical and numerical solutions of linear equations and introduce the concept of convergence. |
| **CO3** | Use Simplex methods and its variants to solve linear programming problems. |
| **CO4** | Analyze the validity of solutions of linear programming problems to changes in parameters. |

**Text Book(s)**

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| --- | --- |
| No | Author(s), Title, Edition, Publishing House |
| T1 | Erwin Kreyszig,Advanced Engineering Mathematics, Wiley India, 9th Edition 2011. |
| T2 | Hamdy A Taha, Operations Research: An Introduction, Pearson/PHI, 8th Edition, 2009. |

**Reference Book(s) & other resources**

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| --- | --- |
| No | Author(s), Title, Edition, Publishing House |
| R1 | B. Dubey, Introductory Linear Algebra, Asian Books Pvt Ltd, 2007. |
| R2 | K Hoffman and R Kunze, Linear Algebra, Pearson Education, 2nd Edition,  2005. |
| R3 | Ravindran, Phillips and Solberg, Operations Research: Principles  and Practice,John Wiley, Second Edition, 2000. |

**Content Structure**

**Module Summary: Linear Algebra**

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| No | Title of the Module | **References** |
| M1 | Matrices, System of equations, determinants and inverse of a matrix |  |
| M2 | Vector spaces and Linear transformations |  |
| M3 | Eigenvalues and eigenvectors |  |
| M4 | Numerical Linear Algebra: Gauss elimination and iterative methods for solving linear systems |  |
| M5 | Matrix eigenvalue problems and power method for eigenvalue |  |

**Module Summary: Optimization**

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| **No** | **Title of the Module** | **References** |
| M6 | Modelling with Linear Programming (LP) and Graphical Method of Solution |  |
| M7 | The Simplex Method |  |
| M8 | Big M and Two Phase Methods |  |
| M9 | Special cases of the Simplex Method |  |
| M10 | Sensitivity Analysis of the Simplex method |  |
| M11 | Dual Simplex Method |  |
| M12 | Generalized Simplex Method |  |
| M13 | Post Optimal Analysis |  |

**Learning Outcomes:**

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| **No** | **Learning Outcomes** |
| LO1 | Effectively use matrix algebra tools to analyze and solve systems of linear equations. |
| LO2 | Identify decision variables and formulate a linear maximization / minimization problem subject to linear constraints. |
| LO3 | Choose the variant of Simplex method which would be effective for the given problem. |
| LO4 | Assess the sensitivity of the solutions due to change in the parameters. |

**Part B: Contact Session Plan**

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| **Academic Term** | II Sem,2018 - 19 |
| **Course Title** | LINEAR ALGEBRA & OPTIMIZATION |
| **Course No** | SEWP ZC252 / CSIW ZC132 |

## Course Contents

**Contact Session -1, Module 1: Matrices, System of equations, determinants and inverse of a matrix**

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| **Time** | **Type** | **Description** | **References** |
| Pre- CS-1 | RL1.1, RL 1.2 | Row-reduced echelon form of a matrix  Consistency and inconsistency of linear system of equations, Inverse of a matrix | T1 |
| During CS-1 | CS-1 | Overview of course coverage and handout and review of module 1 , T1: Sec 7.1-7.3 T1: Sec 7.5, 7.8 | T1 |
| Post-CS-1 | HW | T1: Sec 7.1: Q1-Q8, Sec 7.2: Q1-Q10  T1: Sec 7.3: QNos 1, 4, 6-9, 13-16, Sec 7.8: Q1-Q12. | T1 |
| Lab |  |  |  |

**Contact Session -2, Module 2:** **Vector spaces and Linear transformations**

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| **Time** | **Type** | **Description** | **References** |
| Pre-CS-2 | RL 2.1, RL 2.2,  RL 2.3 | Vector space, subspace and span of a set  Linearly dependent and independent sets, basis and dimension  Linear transformation, its rank and nullity | T1 |
| During CS-2 | CS-2 | T1: Sec 7.4, T1: Sec 7.9, R1: Sec 3.2 | T1 |
| Post-CS-2 | HW | R1: Sec 2.3: Q1-Q4, 9, R1: Q.Nos 3, 4, 5 7, 20. | T1 |
| Lab |  |  |  |

**Contact Session -3, Module 3 Eigenvalues and eigenvectors**

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| **Time** | **Type** | **Description** | **Reference** |
| Pre-CS-3 | RL 3.1 | Eigenvalues and eigenvectors | T1 |
| During CS-3 | CS-3 | T1: Sec 8.3 | T1 |
| Post-CS-3 | HW | T1: Sec 8.1: QNos 1, 6, 10-15, 19-25, Sec 8.3: Q9-Q17 | T1 |
| Lab |  |  |  |

**Contact Session -4, Module 4 Numerical Linear Algebra: Gauss elimination and iterative methods for solving linear systems**

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| **Time** | **Type** | **Description** | **References** |
| Pre-CS-4 | RL 4.1  RL 4.2 | Gauss elimination with partial pivoting, Gauss elimination with scaling and partial pivoting  Iterative methods to solve Ax = b | T1 |
| During CS-4 | CS-4 | T1: Sec 20.1, T1: Sec 20.3, 20.8 | T1 |
| Post-CS-4 | HW | T1: Sec 20.1: Q4-Q14, Sec 20.3: Q3 –Q8, 12, 13, 14  T1: Sec 20.7: QNos 1, 2, 3, 6. Sec 20.8: Q1-Q7. | T1 |
| Lab |  |  |  |

**Contact Session -5, Module 5 Matrix eigenvalue problems and power method for eigenvalue**

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| **Time** | | **Type** | **Description** | **References** |
| Pre-CS-5 | | RL 5.1 | Matrix eigenvalue problems and Power method for finding dominant eigenvalue | T1 |
| During CS-5 | | CS-5 | T1: Sec 20.8 | T1 |
| Post-CS-5 | | HW | T1: Sec 20.7: QNos 1, 2, 3, 6. Sec 20.8: Q1-Q7. | T1 |
| Lab | |  |  |  |
| **Syllabus for Mid-Semester Test: Topics covered in the first 8 contact session** | | | | |

**Contact Session -6, Module 6 Modelling with Linear Programming (LP) and Graphical Method of Solution**

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| **Time** | **Type** | **Description** | **References** |
| Pre-CS-6 | RL 6.1, 6.2 and RL 6.3 | Sec. 2.1 Introduction to formulation with LP, Examples 2.1-1 and 2.1-2, Properties of Linear Programming Problems (LPP)  Sec. 2.2 Graphical Solution, Solution of the Maximization Model, Example 2.2-1  Sec. 2.3.3 Investment Option | T2 |
| During CS-6 | CS-6 | Example 2.1-3, Problem 2.1A 3,4; Section 2.2.2; Problem 2.2A 2,7,8 | T2 |
| Post-CS-6 | HW | Problem 2.1A 1,2; Problem 2,2A 9; Problem 2.3C 1 | T2 |
| Lab |  |  |  |

**Contact Session -7, Module 7: The Simplex Method**

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| Time | Type | Description | Content Reference |
| Pre-CS-7 | RL 7.1 and RL 7.2 | Section 3.1 LP in standard form, Problem 3.1A 4, Problem 3.1B 2; Section 3.2 Transition from Graphical to Algebraic Solution, Example 3.2-1  Section 3.3 The Simplex Method, Examples 3.3-2 and 3.3-3. | T2 |
| During CS-7 | CS-7 | Problem 3.1A 5,6; Problem 3.1B 1,2,4; Problem 3.3B 2,5 | T2 |
| Post-CS-7 | HW | Problem 3.2A 1,3; Problem 3.3B 3,4,6 | T2 |
| Lab |  |  |  |

**Contact Session -8, Module 8: Big M and Two Phase Methods**

**Module 9: Special Cases of Simplex Method and Consistency**

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| **Time** | **Type** | **Description** | **References** |
| Pre-CS-8 | RL 8.1,  RL 8.2,  RL 9.1 | Section 3.4.1 Big M Method, Example 3.4-1  Section 3.4.2 The Two Phase Method, Example 3.4-2  Section 3.5.1 Degeneracy, Example 3.5-1; Section 3.5.2 Alternative Optima, Example 3.5-2; Section 3.5.3 Unbounded Solution, Example 3.5-3 | T2 |
| During CS-8 | CS-8 | Problem 3.4A 1(a), 1(b), 3; Problem 3.4B 2,4, 7 | T2 |
| Post-CS-8 | HW | Problem 3.4A 1(c), 1(d), 4; Problem 3.4B 3,5  Problem 3.5B 3; Problem 3.5C 5; Problem 3.5D 3 | T2 |
| Lab |  |  |  |

**Contact Session -9,**

**Module 10 Sensitivity Analysis in Simplex Method**

**Module 11 The Dual Simplex Method**

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| **Time** | **Type** | **Description** | **Reference** |
| Pre-CS-9 | RL 10.1  RL 11.1 | Section 3.6.1 The Graphical Sensitivity Analysis, Examples 3.6-1 and 3.6-2  Section 4.1 Connection of Primal to Dual, Examples 4.1-1 and 4.1-2; Section 4.2 Primal Dual Relationship, Example 4.2-1 | T2 |
| During CS-9 | CS-9 | Problem 3.5A 2; Problem 3.5B 2; Problem 3.5C 2, 3, 5  Section 3.6.2, Problem 3.6C 3,5  Problem 4.1A 4(a), 4(b); Problem 4.2A 1(a); Problem 4.2C 2; Problem 4.2D 3 | T2 |
| Post-CS-9 | HW | Problem 3.6A 1, Problem 3.6B 2, Problem 3.6C 1  Problem 4.1A 4(c); Problem 4.2A 6; Problem 4.2C 1; Problem 4.2D 3 | T2 |
| Lab |  |  |  |

**Contact Session -10, Module 12 The Generalized Simplex Algorithm**

**Module 13 Post Optimal Analysis**

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| **Time** | **Type** | **Description** | **References** |
| Pre-CS-10 | RL 12.1  RL 13.1 | Section 4.4.1 The Dual Simplex Algorithm, Examples 4.4-1 and 4.4-2; Section 4.4.2  Section 4.5 Post Optimal Analysis, Example 4.5-1 | T2 |
| During CS-10 | CS-10 | Problem 4.4A 2(a), 2(d); Problem 4.4B 1  Problem 4.5A 2(a), 4(i), 4(ii); Section 4.5.2 Changes affecting optimality | T2 |
| Post-CS-10 | HW | Problem 4.4A 1, 2(b), 2(c), 4(b); Problem 4.4B 2  Problem 4.5A 2(b), 3; Problem 4.5B 1; Problem 4.5D 2 | T2 |
| Lab |  |  |  |
| **Syllabus for Comprehensive Exam (Open Book) All topics given in Plan** | | | |